Center of Mass

AP Physics C

Mrs. Coyle

Center of Mass

- The point of an object at which all the mass of the object is thought to be concentrated.
- Average location of mass.

Experimental Determination of CM

- Suspend the object from two different points of the object.
- Where two vertical lines from these two points intersect is the CM.

Location of Center of Mass

The CM could be located:

- within the object (human standing straight)

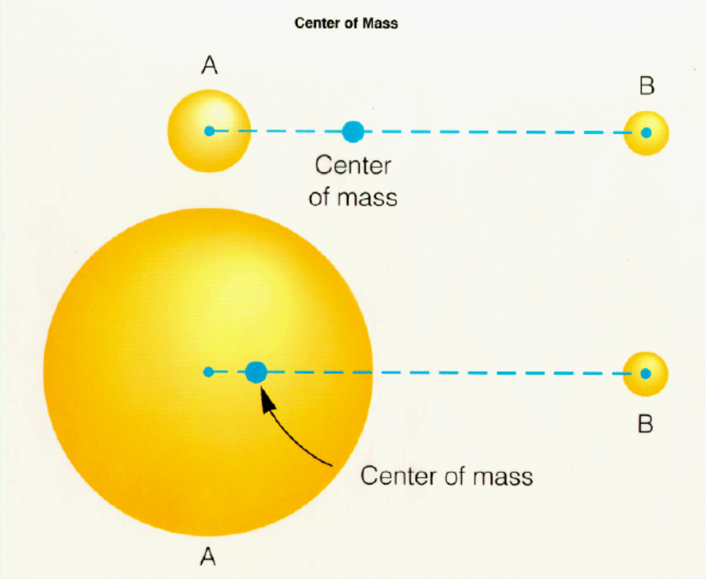
- outside the object (high jumper as she goes over the bar)

Center of Mass is outside the object.

Center of Gravity

- The point of the object where the force of gravity is thought to be acting.
- Average location of weight.
- If g is the same throughout the object then the CM coincides with the CG.

Center of Mass of:



- System of Particles

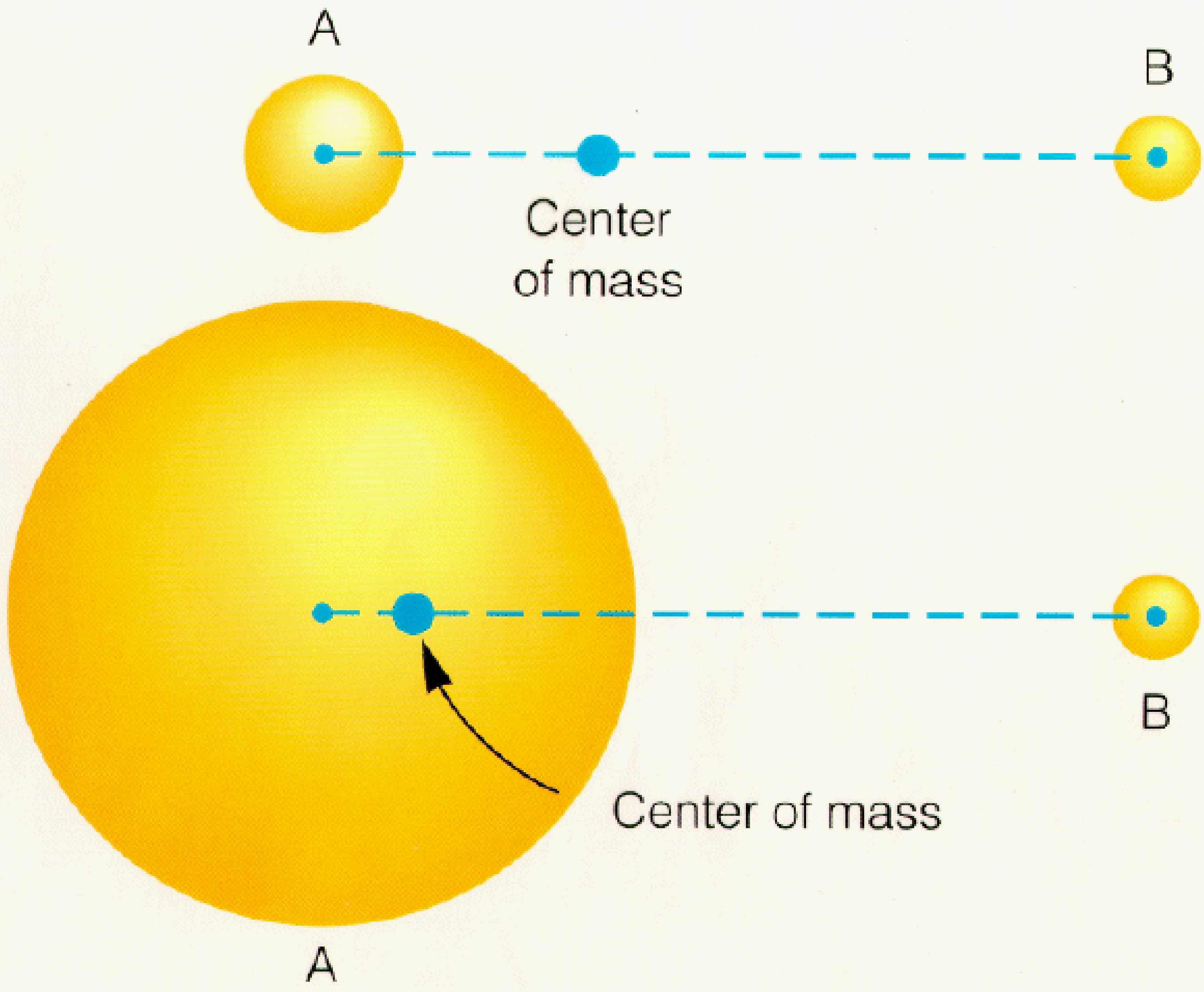
- Extended Object

Center of Mass of a **System of Particles in one Dimension**

$$X_{CM} = \frac{\Sigma m_i x_i}{M}$$

- m_i is the mass of each particle
- x_i is the position of each particle with respect to the origin
- M is the sum of the masses of all particles

Center of Mass



Example 1: Center of Mass in one Dimension

- Find the CM of a system of four particles that have a mass of 2 kg each. Two are located 3cm and 5 cm from the origin on the + x-axis and two are 2 and 4 cm from the origin on the – x-axis

- Answer: 0.5cm

Coordinates of Center of Mass of a System of Particles in Three Dimensions

$$x_{\mathrm{CM}} = \frac{\sum_i m_i x_i}{M} \qquad y_{\mathrm{CM}} = \frac{\sum_i m_i y_i}{M} \qquad z_{\mathrm{CM}} = \frac{\sum_i m_i z_i}{M}$$

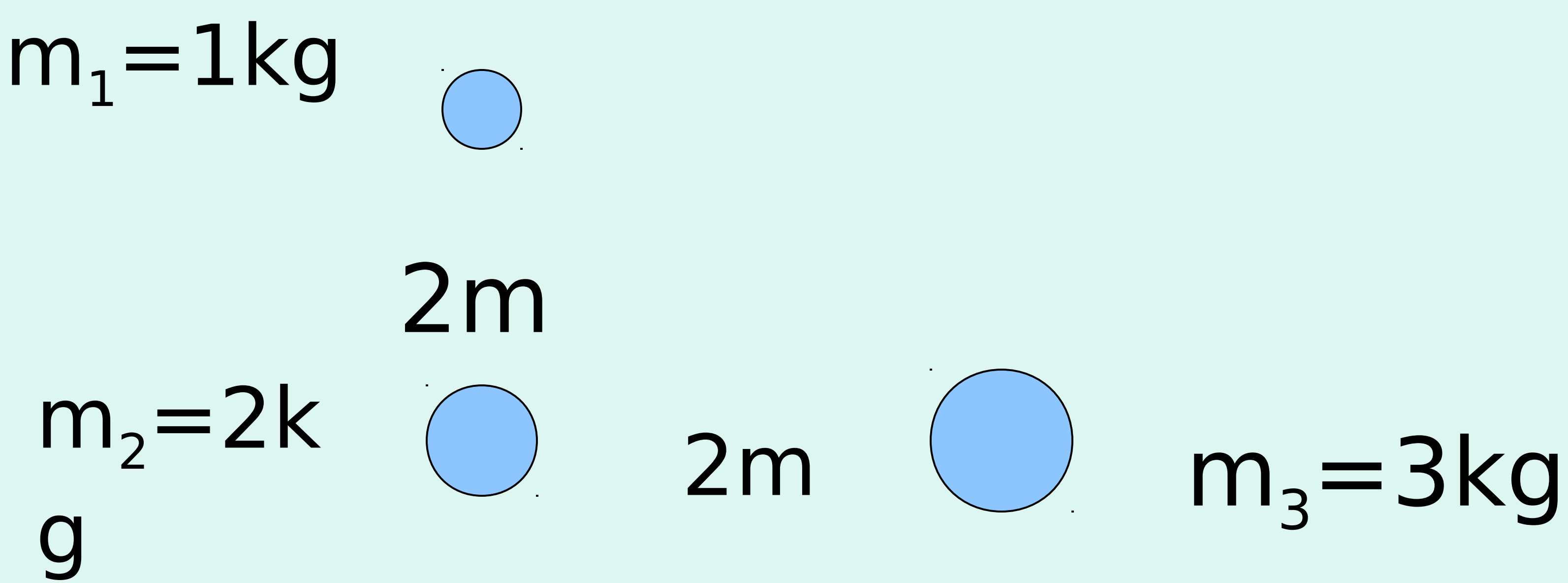
Coordinate of CM using the Position Vector, r

$$\mathbf{r}_{\mathrm{CM}} = \frac{\sum_i m_i \mathbf{r}_i}{M}$$

$$\mathbf{r}_i = x_i \hat{\mathbf{i}} + y_i \hat{\mathbf{j}} + z_i \hat{\mathbf{k}}$$

Example 2: Center of Mass in two Dimensions

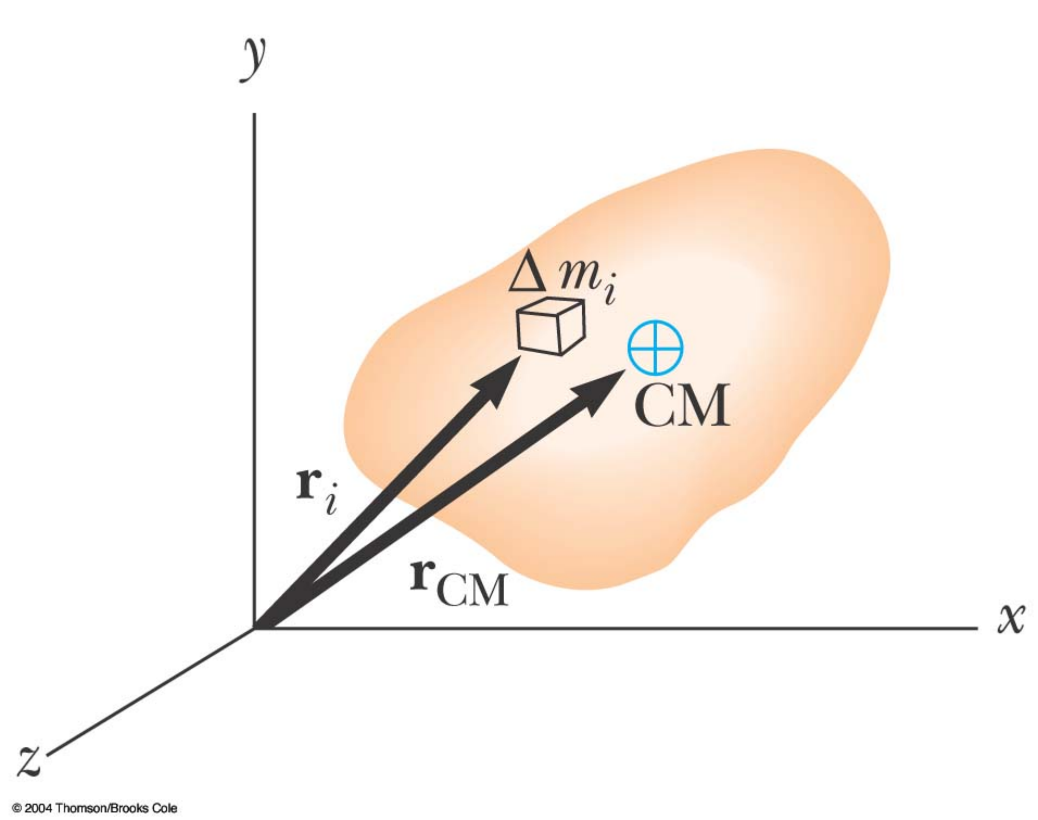
Find the CM of the following system:



Ans: x=1m, y=0.33m

Center of Mass of an Extended Object

An extended object can be considered a distribution of small mass elements, Δm.



Center of Mass of an Extended Object using Position Vector

- Position of the center of mass:

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{M} ? \, dm$$

Center of Mass of an Extended Object

$$x_{\mathrm{CM}} = \frac{1}{M}\int x\,dm \qquad y_{\mathrm{CM}} = \frac{1}{M}\int y\,dm$$

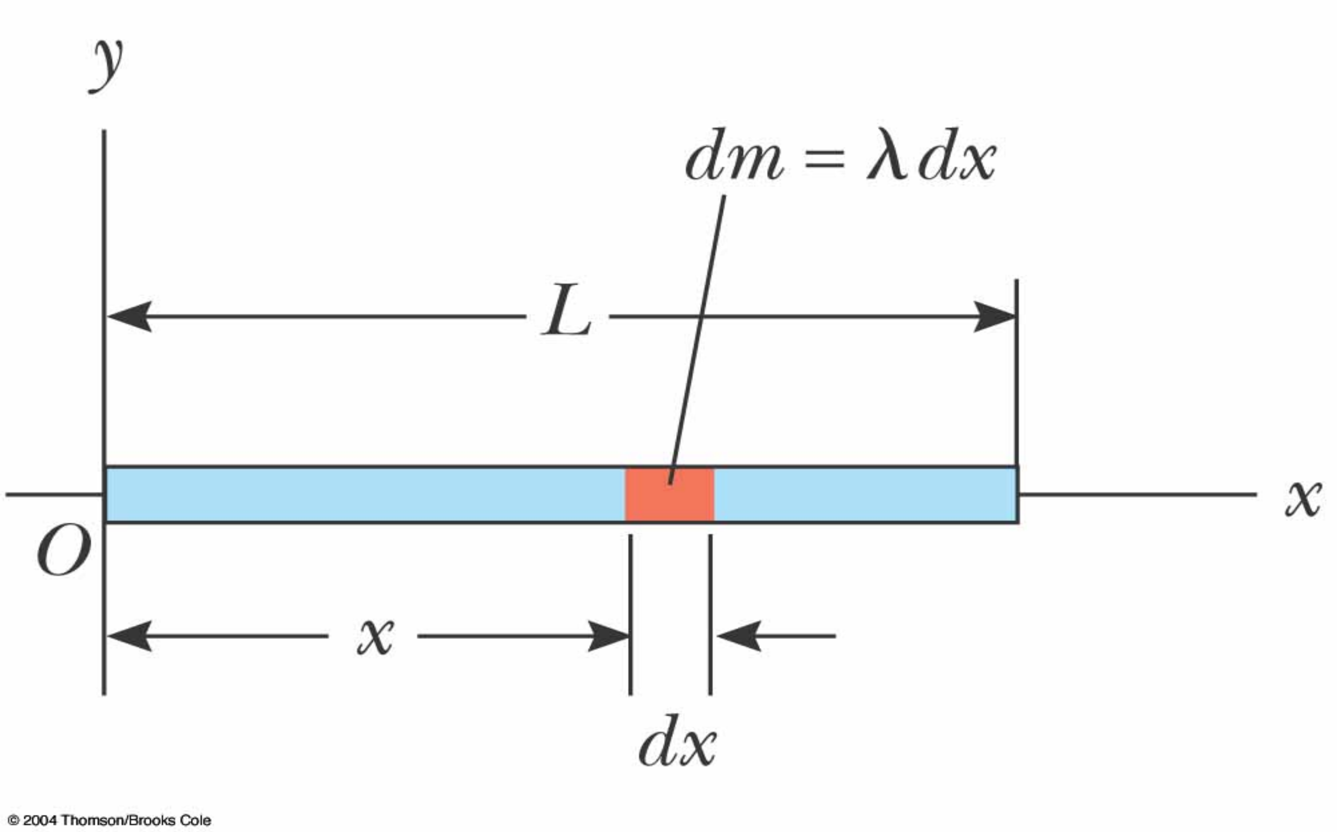
$$z_{\mathrm{CM}} = \frac{1}{M}\int z\,dm$$

CM of Uniform Objects

- **Uniform density**, $\rho = m/V = dm/dV$
- **Uniform mass per unit length**,
  $\lambda = m/x = dm/dx$

Center of Mass of a Rod

- Find the center of mass of a rod of mass *M* and length *L*.



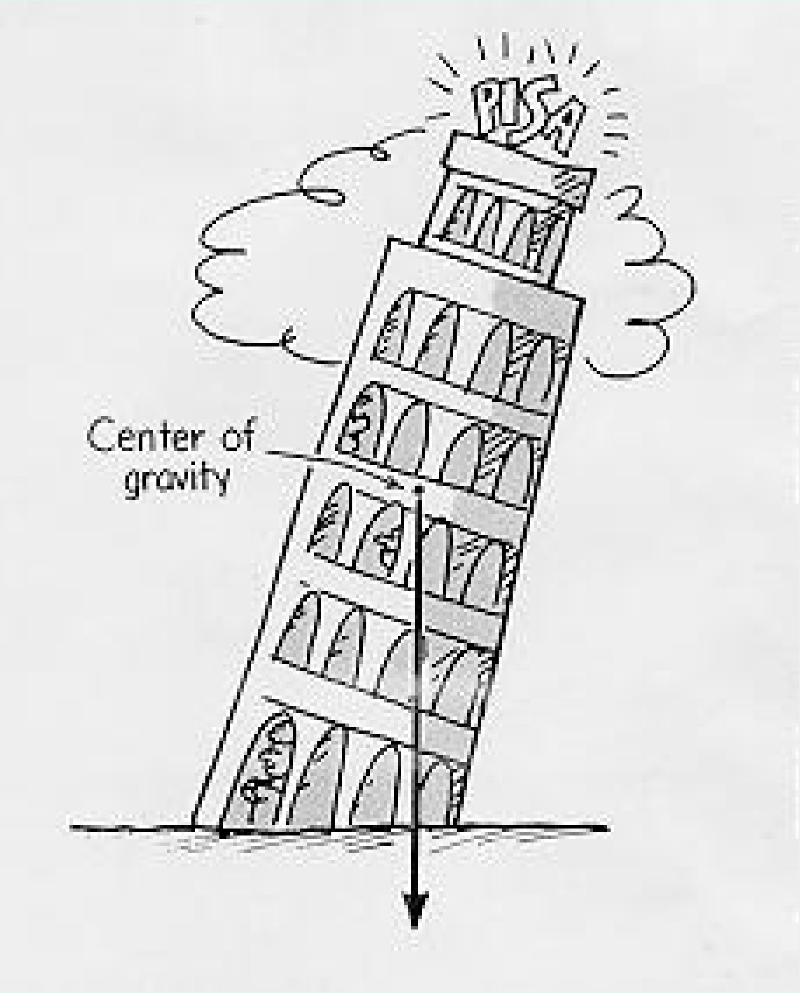
Ans: $x_{CM} = L / 2$, (or $y_{CM} = z_{CM} = 0$)

CM of Symmetrical Object

- The CM of any symmetrical object lies on an axis of symmetry and on any plane of symmetry.

Toppling Rule of Thumb

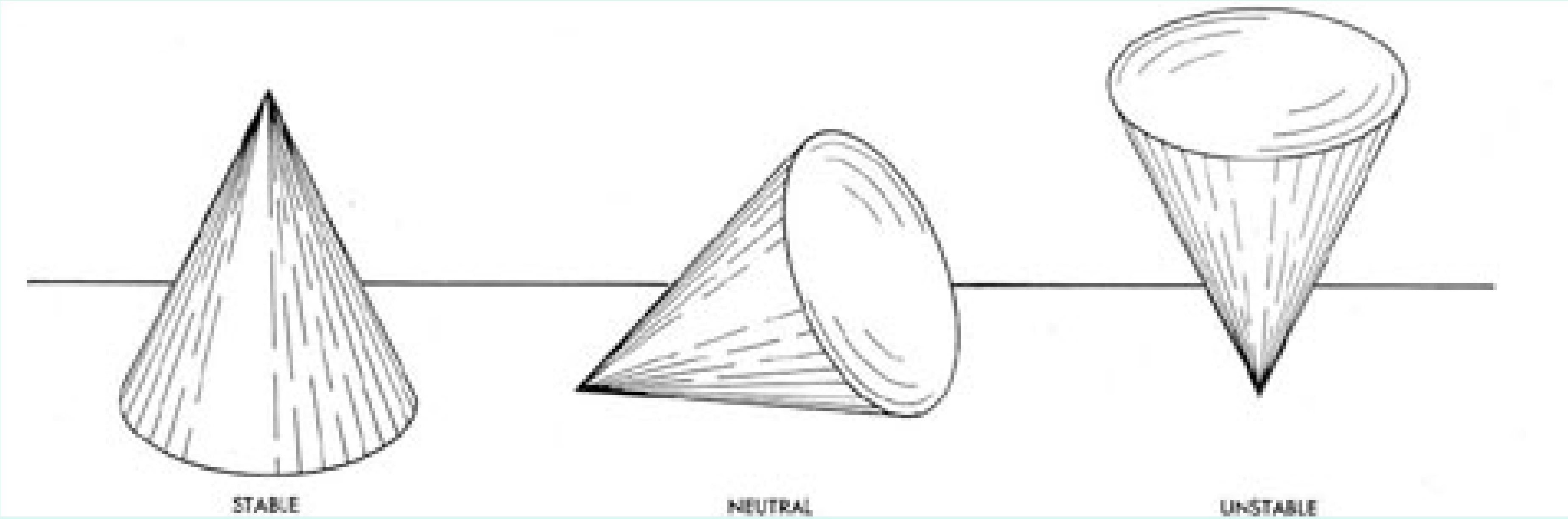
- If the CG of the object is above the area of support, the object will remain upright.
- If the CG is outside the area of support the object will topple.



Another look at Stability

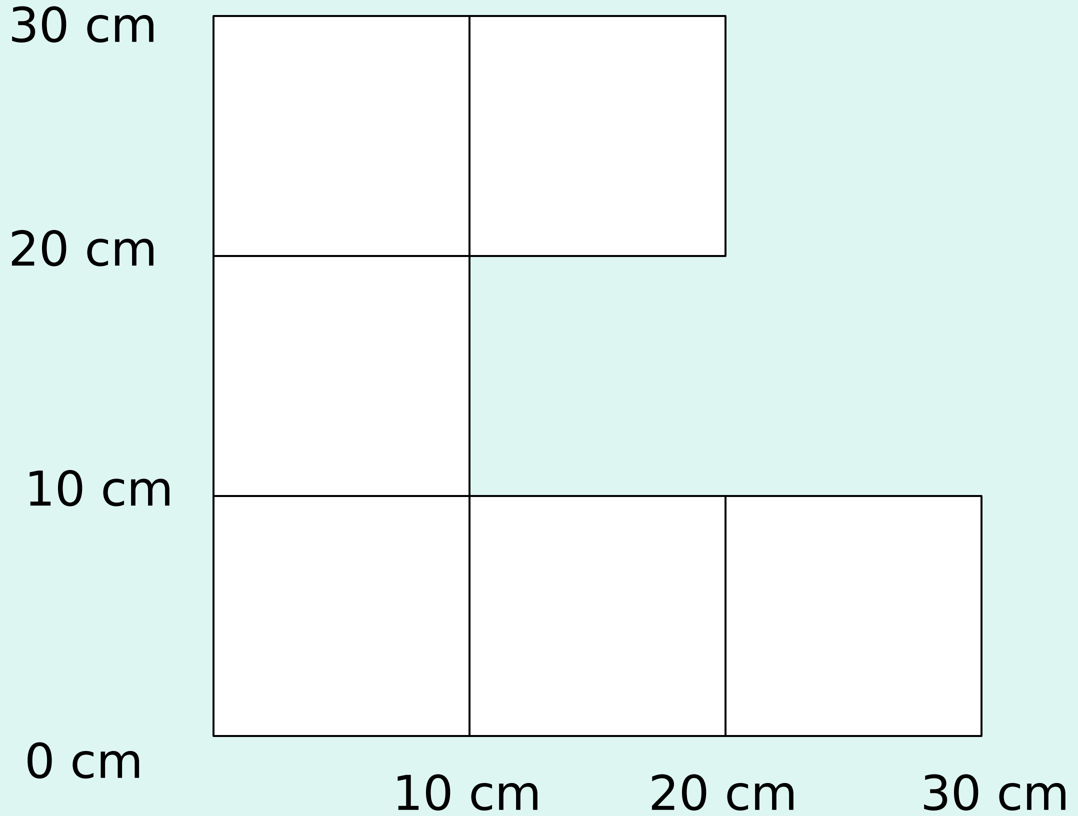
- **Stable equilibrium:** when for a balanced object a displacement raises the CG (to higher U so it tends to go back to the lower U).
- **Unstable equilibrium:** when for a balanced object a displacement lowers the CG (lower U).
- **Neutral equilibrium:** when the height of the CG does not change with displacement.

Stability



Example #41

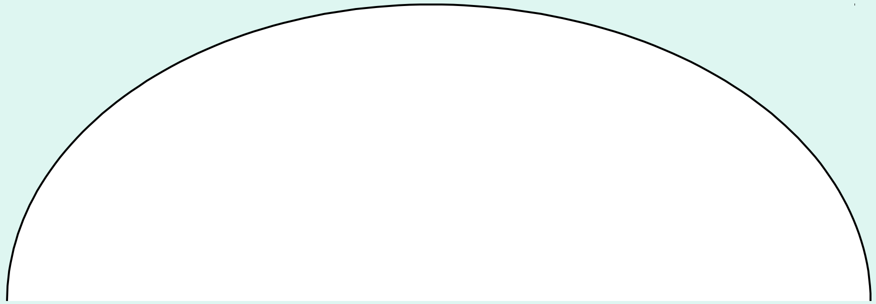
A uniform piece of sheet steel is shaped as shown. Compute the x and y coordinates of the center of mass.



Ans:
x=11.7cm,
y=13.3cm

Example #44 "Fosbury Flop"

- Find the CM



Ans: 0.0635L below the top of the arch